MENIIT
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JEE MAINS-2016

10-04-2016 (Online)

IMPORTANT INSTRUCTIONS

1. The test is of **3** hours duration.
2. The Test Booklet consists of **90** questions. The maximum marks are **360**.
3. There are **three** parts in the question paper A, B, C consisting of **Physics**, **Chemistry and Mathematics and** having 30 questions in each part of equal weightage. Each question is allotted **4 (four)** marks for each correct response.
4. Candidates will be awarded marks as stated above in instruction No.3 for correct response of each question. 1/4 (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
5. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 4 above.

PART-A-PHYSICS

1. A, B, C and D are four different physical quantities having different dimensions. None of them is dimensionless. But we know that the equation AD=C ln(BD) holds true. Then which of the combination is not a meaningful quantity?

(1) $A^2 - B^2C^2$ (2*) $\frac{(A-C)}{D}$ (3) $\frac{A}{B} - C$ (4) $\frac{C}{BD} - \frac{AD^2}{C}$

Sol. AD = Cℓn (BD)
(B) (D) $\rightarrow$ dimensionless

[AD] = [C]

Checking options one by one

$\frac{A-C}{D}$ dimensions of A are not same as dimensions of C.

meaningless

2. A particle of mass M is moving in a circle of fixed radius R in such a way that its centripetal acceleration at time t is given by n^2 R t^2 where n is a constant. The power delivered to the particle by the force acting on it, is :

(1*) M n^2R^2t (2) M nR^2t (3) M nR^2t^2 (4) 1/2 $Mn^2R^2t^2$

Sol. $\frac{V^2}{R} = n^2Rt^2$

$\Rightarrow V^2 = n^2R^2t^2$

$\Rightarrow V = nRt$

$\Rightarrow \frac{dV}{dt} = nR$

$P = F_tV$

$= \frac{mdV}{dt}V$

= mnR, nRt

$P = n^2R^2t\, m$

3. Concrete mixture is made by mixing cement, stone and sand in a rotating cylindrical drum. If the drum rotates too fast, the ingredients remain stuck to the wall of the drum and proper mixing of ingredients does not take place. The maximum rotational speed of the drum in revolutions per minute(rpm) to ensure proper mixing is close to :

(Take the radius of the drum to be 1.25 m and its axle to be horizontal) :

(1) 0.4 (2) 1.3 (3) 8.0 (4*) 27.0

Ans. Bonus

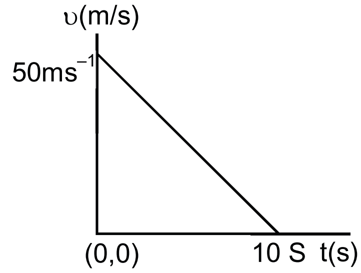
Sol. $\varsigma \geq \sqrt{5gr}$

$\omega = \frac{v}{r} \leq \sqrt{\frac{5g}{r}} = \sqrt{\frac{50 \times 8}{10}} = \sqrt{\frac{400}{10}}$

$\omega = \sqrt{40}$ rad / s

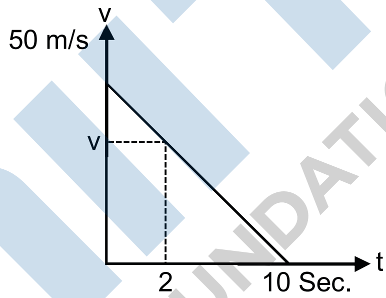
$= \dfrac{60\sqrt{10}}{\pi}$ rpm

4. Velocity-time graph for a body of mass 10 kg is shown in figure. Work-done on the body in first two seconds of the motion is :



(1) 12000 J (2) – 12000 J (3*) – 4500 J (4) – 9300 J

Sol. $\dfrac{8}{V} = \dfrac{10}{50}$



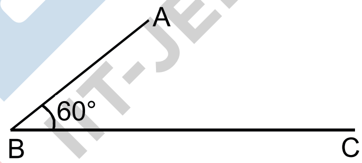
V = 40 m/s

$W = \dfrac{1}{2} \times 10 \times (1600 - 2500)$

$= \dfrac{1}{2} \times 10 \times (-900)$

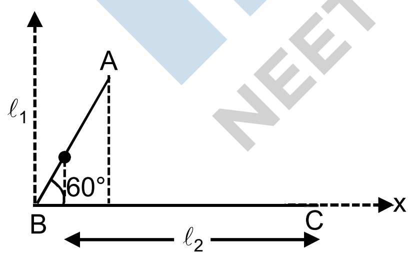
=–4500 J

5. In the figure shown ABC is a uniform wire. If centre of mass of wire lies vertically below point A, then $\dfrac{BC}{AB}$ is close to :



(1) 1.85 (2*) 1.37 (3) 1.5 (4) 3

Sol.



If CM lies vertically below A ⇒ as per choose coordinate axis in x-coordinate is equal to $\dfrac{\ell_1}{2}$

$$X_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

$$\frac{\ell_1}{2} = \frac{(\lambda\ell_1)\left(\frac{\ell_1}{4}\right) + (\lambda\ell_2)\left(\frac{\ell_2}{2}\right)}{\lambda(\ell_1 + \ell_2)}$$

$$\frac{\ell_1^2}{2} = \frac{\ell_1\ell_2}{2} = \frac{\ell_1^2}{4} + \frac{\ell_2^2}{2}$$

$$\frac{\ell_1^2}{4} + \frac{\ell_1\ell_2}{2} - \frac{\ell_2^2}{2} = 0$$

$$\ell_1^2 + 2\ell_1\ell_2 - 2\ell_2^2 = 0$$

$$\ell_1 = \frac{-2\ell_2 \pm \sqrt{4\ell_1^2 + 4.1(+2\ell_2^2)}}{2}$$

$$\ell_1 = \frac{-2\ell_2 + \sqrt{12\ell_2^2}}{2}$$

$$\ell_1 = \frac{-2\ell_2 + 2\sqrt{3}\ell_2}{2}$$

$$\ell_1 = (\sqrt{3} - 1)\ell_2$$

$$\frac{\ell_2}{\ell_1} = \frac{1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$\frac{\ell_2}{\ell_1} = \frac{\sqrt{3}+1}{2} = \frac{2.732}{2} = 1.366$$

$\simeq$ 1.37

6. An astronaut of mass m is working on a satellite orbiting the earth at a distance h from the earth's surface. The radius of the earth is R, while its mass is M. The gravitational pull F_G on the astronaut is :

(1) Zero since astronaut feels weightless

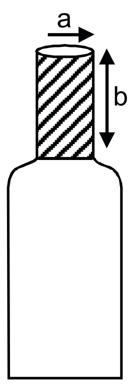
(2) $0 < F_G < \frac{GMm}{R^2}$

(3) $\frac{GMm}{(R+h)^2} < F_G < \frac{GMm}{R^2}$

(4*) $F_G = \frac{GMm}{(R+h)^2}$

Sol. $F_G = \frac{GMm}{(R+h)^2}$

7. A bottle has an opening of radius a and length b. A cork of length b and radius (a + Δa) where (Δa << a) is compressed to fit into the opening completely (See figure). If the bulk modulus of cork is B and frictional coefficient between the bottle and cork is μ then the force needed to push the cork into the bottle is :



(1) $(\pi\mu B\, b)\,\Delta a$

(2) $(2\pi\mu B\, b)\,\Delta a$

(3) $(\pi\mu B\, b)\, a$

(4*) $(4\pi\mu B\, b)\,\Delta a$

Sol. $\beta\frac{\Delta Vg}{V} = -\Delta P\,]$

$V_i = \pi(a + \Delta a)^2 b$

$V_f = \pi a^2 b$

$\Delta V \approx -2\pi\, ab\Delta a$

$\frac{\Delta V}{V} = \frac{-2\pi ab\ \Delta a}{\pi a^2 b} = \frac{-2\Delta a}{a}$

$\Rightarrow \Delta P = \frac{2\beta\Delta a}{a}$

Normal force $= \frac{2\beta\Delta a}{a} 2\pi\alpha b$

$= 4\pi\beta\ b\ \Delta\ a$

Friction = μN

$= 4\pi\ \mu\beta\ b\ \Delta\ a$

8. A Carnot freezer takes heat from water at 0°C inside it and rejects it to the room at a temperature of 27°C. The latent heat of ice is 336×10^3 J kg^{-1}. If 5 kg of water at 0°C is converted into ice at 0°C by the freezer, then the energy consumed by the freezer is close to :

(1*) 1.67×10^5 J (2) 1.68×10^6 J (3) 1.51×10^5 J (4) 1.71×10^7 J

Sol. heat required to freeze 5 kg water

$= 5 \times 336 \times 10^3$

$= 1680 \times 10^3$ Joule

$\Rightarrow Q_1 = 1680$ KJ

for carnot's cycle

$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$

$\frac{Q_2}{1680} = \frac{300}{273}$

$Q_2 = 1680 \times \frac{300}{273}$ KJ

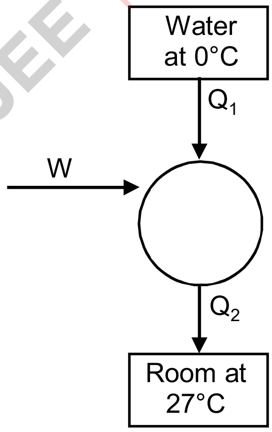
$W = Q_2 - Q_1$

$= 1680\left(\frac{300}{273} - 1\right)$

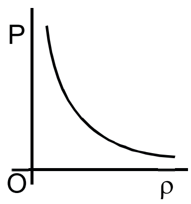
$= \frac{1680 \times 27}{273} \times 10^3$ J

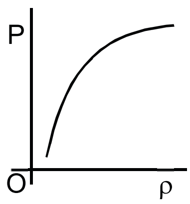
$= 166.15 \times 10^3$J

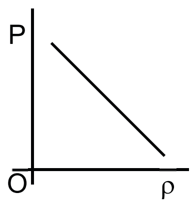
$= 1.66 \times 10^5$ KJ

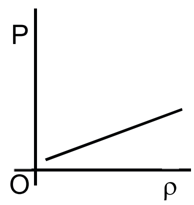


9. Which of the following shows the correct relationship between the pressure 'P' and density ρ of an ideal gas at constant temperature?

(1)


(2)


(3)


(4*)


Sol. $\rho = \dfrac{PM}{RT}$

$P = \dfrac{\rho RT}{M}$

$P \propto \rho$

10. In an engine the piston undergoes vertical simple harmonic motion with amplitude 7 cm. A washer rests on top of the piston and moves with it. The motor speed is slowly increased. The frequency of the piston at which the washer no longer stays in contact with the piston, is close to :

(1) 0.1 Hz (2) 1.2 Hz (3) 0.7 Hz (4*) 1.9 Hz

Sol. $\omega^2 A = g$ $\quad T = \dfrac{2\pi}{\omega}$ $\quad \omega = 2\pi t$

$\omega^2(0.07) = 10\ m/s^2$

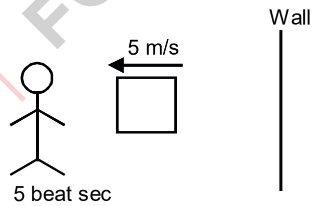
$4\pi^2\ r^2\ (0.07) = 10\ m/s^2$

$t = \dfrac{5}{\sqrt{7}} Hz = 1.9\ Hz$

11. A toy-car, blowing its horn, is moving with a steady speed of 5 m/s, away from a wall. An observer, towards whom the toy car is moving, is able to hear 5 beats per second. If the velocity of sound in air is 340 m/s, the frequency of the horn of the toy car is close to :

(1) 680 Hz (2) 510 Hz (3) 340 Hz (4*) 170 Hz

Sol. $F_{dir} = \left(\dfrac{340}{340-5}\right)f$



$f_{ind} = \left(\dfrac{340}{340+5}\right)f$

$f_{ind} - f_{dir} = 5$

$\left\{\left(\dfrac{340}{340+5}\right)\left(\dfrac{340}{340+5}\right)\right\}f = 5$

$340\left\{\dfrac{10}{(300-5(340+5))}\right\} = 5$

$f = \dfrac{340 \times 5}{10}$

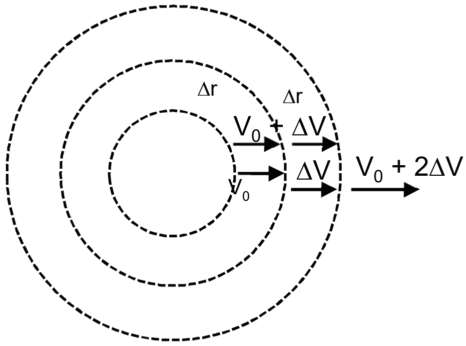
f = 170 Hz

12. Within a spherical charge distribution of charge density $\rho(r)$, N equipotential surfaces of potential V_0, $V_0 + \Delta V$, $V_0 + 2\Delta V$, ... $V_0 + N\Delta V$ ($\Delta V > 0$), are drawn and have increasing radii r_0, r_1, r_2,..........r_N, respectively. If the difference in the radii of the surfaces is constant for all values of V_0 and ΔV then :

(1) $\rho(r)\ \alpha\, r$ (2) $\rho(r)$ = constant (3*) $\rho(r)\ \alpha\ \dfrac{1}{r}$ (4) $\rho(r)\ \alpha\ \dfrac{1}{r^2}$

Sol. $\frac{\Delta V}{\Delta r} \to \text{constant}$

$\Rightarrow$ uniform E. field.



$(E)(4\pi r)^2 = \frac{1}{\varepsilon_0}\int \rho dV$

$(E)(4\pi r)^2 = \frac{1}{\varepsilon_0}\int_0^r \rho 4\pi r^2 dr$

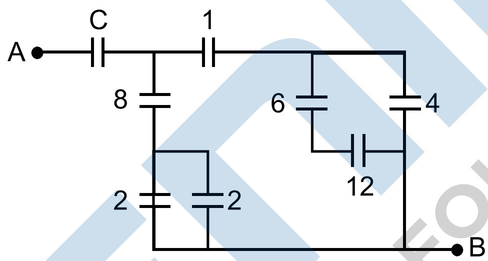
$(E)(4\pi r)^2 = \frac{1}{\varepsilon_0}\int \rho 4\pi r^2 d\,r$

After integral on RHS

We must obtain r^2

$\Rightarrow \rho \propto \frac{1}{r}$

13. Figure shows a network of capacitors where the numbers indicates capacitances in micro Farad. The value of capacitance C if the equivalent capacitance between point A and B is to be 1 μF is :



(1) $\frac{31}{23}\mu F$ (2*) $\frac{32}{23}\mu F$ (3) $\frac{33}{23}\mu F$ (4) $\frac{34}{23}\mu F$

Sol. $\frac{8\times12}{18} = 4\mu F$

$4\mu F + 4\mu F = 8\mu F$

$\frac{8\times1}{8+1} = \frac{8}{9}\mu F$

$\frac{8\times4}{8+4} = \frac{32}{12} = \frac{8}{3}\mu F$

$\frac{8}{3}+\frac{8}{9} = \frac{24+8}{9} = \frac{32}{9}\times C = \frac{32}{9}+C$

$\Rightarrow C\left(\frac{32-9}{9}\right) = \frac{32}{9}$

$C = \frac{32}{23}\mu F$

14. The resistance of an electrical toaster has a temperature dependence given by $R(T) = R_0[1 + \alpha(T - T_0)]$ in its range of operation. At T_0 = 300 K, R = 100 Ω and at T = 500 K , R = 120 Ω. The toaster is connected to a voltage source at 200 V and its temperature in raised at a constant rate from 300 to 500 K in 30 s. The total work done in raising the temperature is :

(1*) 400 $\ell n \frac{1.5}{1.3}$ J (2) 200 $\ell n \frac{2}{3}$ J (3) 400 $\ell n \frac{5}{6}$ J (4) 300 J

Sol. $\frac{(200)^2}{R_0\left(1+\alpha\left(T-T_0\right)\right)}$

$T \rightarrow$ temperature at 't'

$T_0 \rightarrow$ temperature at t = 300 K

$T - T_0 = \frac{200}{30}t$

$T - T_0 = \frac{20t}{30}$

$\int_0^{30} \frac{(200)^2}{100\left(1+\alpha\frac{20t}{3}\right)}dt = \frac{200 \times 200}{100}\int_0^{30}\frac{dt}{1+\frac{20\alpha}{3}t}$

$= \frac{400 \times 3}{20\alpha}\ell n\left(\frac{\frac{1+20\alpha}{3} \times 30}{1}\right)$

120 = 100 (1 + α(200))

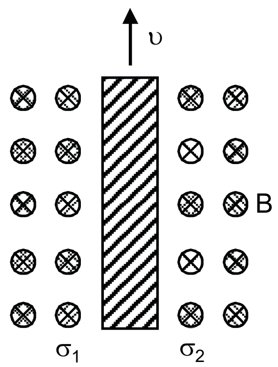
$1 + (200)\alpha = \frac{6}{5}$

$(200\alpha) = \frac{1}{5}$

$\alpha = \frac{1}{1000}$

$= 60{,}000\ \ell n\left(\frac{6}{5}\right)$

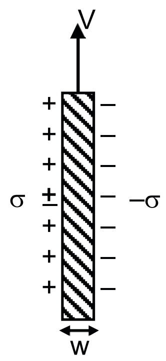
15. Consider a thin metallic sheet perpendicular to the plane of the paper moving with speed 'v' in a uniform magnetic field B going into the plane of the paper (see figure). If charge densities σ_1 and σ_2 are induced on the left and right surfaces, respectively, of the sheet then (ignore fringe effects) :



(1) $\sigma_1 = \varepsilon_0 v B$, $\sigma_2 = -\varepsilon_0 vB$ (2*) $\sigma_1 = \frac{\varepsilon_0 vB}{2}$, $\sigma_2 = \frac{-\varepsilon_0 vB}{2}$

(3) $\sigma_1 = \sigma_2 = \varepsilon_0 vB$ (4) $\sigma_1 = \frac{-\varepsilon_0 vB}{2}$, $\sigma_2 = \frac{\varepsilon_0 vB}{2}$

Sol. direction of V × B is towards left therefore induced change dentition will be



electric field $\frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$

$\frac{\sigma}{2\varepsilon_0} \times W = (B)(V)(\omega)$

$\sigma = BV\varepsilon_0$

16. A fighter plane of length 20 m, wing span (distance from tip of one wing to the tip of the other wing) of 15 m and height 5m is flying towards east over Delhi. Its speed is 240 ms^{-1}. The earth's magnetic field over Delhi is 5×10^{-5} T with the declination angle ~ 0° and dip of θ such that $\sin\theta = \frac{2}{3}$. If the voltage developed is V_B between the lower and upper side of the plane and V_W between the tips of the wings then V_B and V_W are close to :

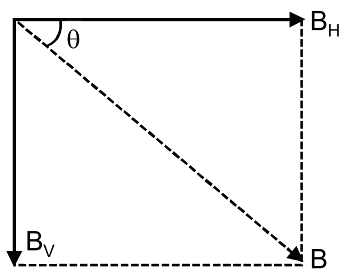
(1) V_B = 45 m V; V_W = 120 mV with right side of pilot at higher voltage

(2*) V_B = 45 mV; V_W = 120 mV with left side of pilot at higher voltage

(3) V_B = 40mV; V_W = 135 mV with right side of pilot at high voltage

(4) V_B = 40 mV; V_W = 135 mV with left side of pilot at higher voltage

Sol. $V_B = B_4$ (5) (240)



$B_H = B\cos\theta$

$B_H = \frac{5\sqrt{5} \times 10^{-5}}{3}$

$B_V = \frac{10}{3} \times 10^{-5}$T

$V_B = \frac{5\sqrt{5}}{3} \times 10^{-5} \times 5 \times 240$

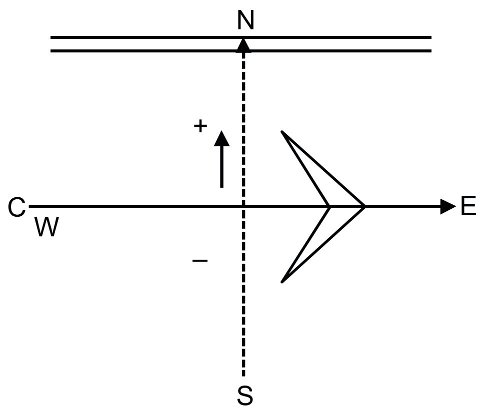
V_B = 44.6 mV = 45 mV

$V_W = B_V \ell V$

$= 10^{-4} \times 1200$

V_ω = 120 mV

(left side at fighter voltage)



17. A conducting metal circular-wire-loop of radius r is placed perpendicular to a magnetic field which varies with time as $B = B_0 e^{-t/\tau}$, were B_0 and τ are constants, at time t = 0. If the resistance of the loop is R then the heat generated in the loop after a long time ($t \to \infty$) is :

(1) $\frac{\pi^2 r^4 B_0^4}{2\tau R}$ (2*) $\frac{\pi^2 r^4 B_0^2}{2\tau R}$ (3) $\frac{\pi^2 r^4 B_0^2 R}{\tau}$ (4) $\frac{\pi^2 r^4 B_0^2}{\tau R}$

Sol. heat equated $= \int_0^\infty i^2 R dt$

$$= \int_0^\infty \frac{\varepsilon_{ind^2}}{Rt} R dt$$

$$= \frac{1}{R}\int_0^\infty \varepsilon_{ind^2} dt$$

$$= \frac{1}{R}\frac{\pi^2 r^4 B_0^2}{\tau^2}\int_0^\infty e^{-2t/\tau} dt$$

$$= \frac{1}{R}\frac{\pi^2 r^4 B_0^2}{\tau^2}\left.\frac{e^{-2t/\tau}}{-2/\tau}\right|_0^\infty$$

$$= +\frac{\pi^2 r^4 B_0^2 \tau}{2R\tau^2}\{0+1\}$$

$$\frac{\pi^2 r^4 B_0^2}{2R\tau}$$

18. Consider an electromagnetic wave propagating in vacuum. Choose the correct statement :

(1*) For an electromagnetic wave propagating in + x direction the electric field is $\vec{E} = \frac{1}{\sqrt{2}} E_{yz}(x, t)\ (\hat{y} - \hat{z})$

and the magnetic field is $\vec{B} = \frac{1}{\sqrt{2}} B_{yz},(x,t)(\hat{y} + \hat{z})$

(2) For an electromagnetic wave propagating in + x direction the electric field is $\vec{E} = \frac{1}{\sqrt{2}} B_{yz}(y,z,t)(\hat{y} + \hat{z})$

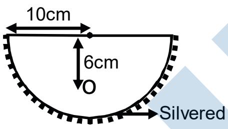
and the magnetic field is $\vec{B} = \frac{1}{\sqrt{2}} B_{yz}(y,z,t)(\hat{y} + \hat{z})$

(3) For an electromagnetic wave propagating in + y direction the electric field is $\vec{E} = \frac{1}{\sqrt{2}} E_{yz}(x,t)\hat{y}$ and the magnetic field is $\vec{B} = \frac{1}{\sqrt{2}} B_{yz}(x,t)\hat{z}$

(4) For an electromagnetic wave propagating in + y direction the electric field is $\vec{E} = \frac{1}{\sqrt{2}} E_{yz}(x,t)\hat{z}$ and the magnetic field is $\vec{B} = \frac{1}{\sqrt{2}} B_{z}(x,t)\hat{y}$

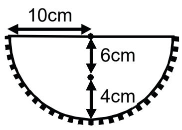
Sol. If wave is propagating in x direction, $\vec{E}$ & $\vec{B}$ must be functions of (x, t) & must be in y-z plane.

19. A hemispherical glass body of radius 10 cm and refractive index 1.5 is silvered on its curved surface. A small air bubble is 6 cm below the flat surface inside it along the axis. The position of the image of the air bubble made by the mirror is seen.



(1) 14 cm below flat surface
(2) 30 cm below flat surface
(3*) 20 cm below flat surface
(4) 16 cm below flat surface

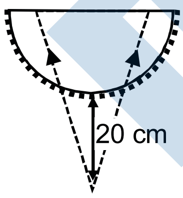
Sol.



$$\frac{1}{V} + \frac{1}{-4} = \frac{1}{-5}$$

$$\frac{1}{V} = \frac{1}{4} - \frac{1}{5} = \frac{5-4}{20}$$

V = 20 cm



I_1 acts as object for plane surface

$$d' = \frac{300}{15} = 20\text{cm}$$

20 cm below plane surface

20. Two stars are 10 light years away from the earth. They are seen through a telescope of objective diameter 30 cm. The wavelength of light is 600 nm. To see the stars just resolved by the telescope, the minimum distance between them should be (1 light year = 9.46×10^{15} m) of the order of :

(1*) 10^6 km
(2) 10^8 km
(3) 10^{11} km
(4) 10^{10} km

Sol. $\theta = \frac{1.22\lambda}{D}$

$$\frac{x}{10 \text{ light year}} = \frac{1.22 \times 60 \times 10^{-9}}{30 \times 10^{-2}}$$

$$\frac{x}{10 \text{ light year}} = 24.4 \times 10^{-7}$$

$x = 2.4 \times 10^{-7} \times 9.46 \times 10^{15}$ m

23.08×10^{6} km

21. A photoelectric surface is illuminated successively by monochromatic light of wavelengths λ and $\frac{\lambda}{2}$. If the maximum kinetic energy of the emitted photoelectrons in the second case is 3 times that in the first case, the work function of the surface is :

(1) $\frac{hc}{3\lambda}$ (2*) $\frac{hc}{2\lambda}$ (3) $\frac{hc}{\lambda}$ (4) $\frac{3hc}{\lambda}$

Sol. $KE_1 = \frac{hc}{\lambda} - W$ .... (i)

$3K.E_1 = \frac{2hc}{\lambda} - W$

$\Rightarrow K.E_1 = \frac{2hc}{3\lambda} - \frac{W}{3}$ .... (ii)

$\Rightarrow \frac{hc}{\lambda} - W = \frac{2hc}{3\lambda} - \frac{W}{3}$

(Equating (i) & (ii))

$\Rightarrow \frac{hc}{\lambda}\left(\frac{1}{3}\right) = \frac{2W}{3}$

$\Rightarrow W = \frac{hc}{2\lambda}$

22. A neutron moving with a speed 'v' makes a head on collision with a stationary hydrogen atom in ground state. The minimum kinetic energy of the neutron for which inelastic collision will take place is :

(1) 10.2 eV (2) 16.8 eV (3) 12.1 eV (4*) 20.4 eV

Sol. Assuming perfectly inelastic collision for maximum loss in K.E.

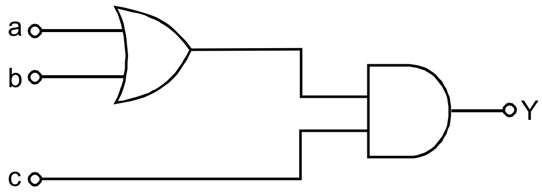
$V_f = \frac{V_i}{2}$

$\Rightarrow$ $K.E_f = \frac{1}{2}(2m) = \frac{V_i^2}{4} \Rightarrow$ $K.E_f = \frac{KE_i}{2}$

$\Rightarrow$ $\text{loss} = \frac{KE_i}{2}$ $\Rightarrow$ $\frac{KE_i}{2} \geq 10.2$ eV

$\Rightarrow$ $K.E_i \geq 20.4$ eV

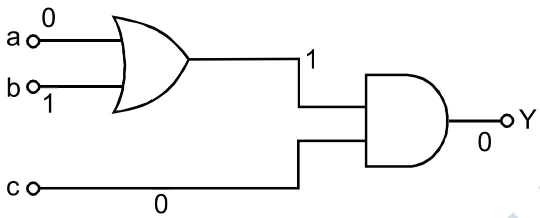
23. To get an output of 1 from the circuit shown in figure the input must be :



(1) a = 0, b = 1, c = 0 (2) a = 1, b = 0, c = 0

(3*) a = 1, b = 0, c = 1 (4) a = 0, b = 0, c = 1

Sol. Checking options one by one



We must note that c = 1

And out put of OR gate must also be 1

therefore

24. A modulated signal C_m (t) has the form $C_m(t) = 30 \sin 300\pi t + 10 (\cos 200\pi t - \cos 400\pi t)$. The carrier frequency f_c, the modulating frequency (message frequency)f_ω, and the modulation index μ are respectively given by :

(1) $f_c = 200$ Hz; $f_\omega = 50$ Hz; $\mu = \frac{1}{2}$ (2*) $f_c = 150$ Hz; $f_\omega = 50$ Hz; $\mu = \frac{2}{3}$

(3) $f_c = 150$ Hz; $f_\omega = 30$ Hz; $\mu = \frac{1}{3}$ (4) $f_c = 200$ Hz; $f_\omega = 30$ Hz; $\mu = \frac{1}{2}$

Sol. ℓ_m (t) = 30 sin (300 πt) + 10 cos (400 πt)

$$f = \frac{\omega}{2\pi} = \frac{300\pi}{2\pi} = 150$$

$$= \frac{20\pi}{2\pi} = 100$$

$$\frac{400\pi}{2\pi} = 200$$

$f_c = 150$ Hz

$f_m = 50$ Hz

$$\mu = \frac{2}{3}$$

25. A particle of mass m is acted upon by a force F given by the empirical law $F = \frac{R}{t^2} v(t)$. If this law is to be tested experimentally by observing the motion starting from rest, the best way is to plot :

(1) v(t) against t^2 (2) log v(t) against $\frac{1}{t^2}$ (3) log v(t) against t (4*) log v(t) against $\frac{1}{t}$

Sol. $m\frac{dV}{dt} = \frac{R}{t^2}V$

$\Rightarrow m\frac{dV}{dt} = R\frac{dt}{t^2}$

$\Rightarrow m\int_{V_1}^{V_2}\frac{dV}{dt} = R\int_{t_1}^{t_2}\frac{dt}{t^2}$

$\Rightarrow m\ell n\left(\frac{V_2}{V_1}\right) = \frac{-R}{t}\Big|_{t_1}^{t_2}$

$\Rightarrow \ell n\left(\frac{V_2}{V_1}\right) = \frac{-R}{m}\left(\frac{1}{t_2} - \frac{1}{t_1}\right)$

logV vs $\frac{1}{t}$ will be a st. line curve

26. A thin 1 m long rod has a radius of 5 mm. A force of 50 πkN is applied at one end to determine its Young's modulus. Assume that the force is exactly known. If the least count in the measurement of all lengths is 0.01 mm, which of the following statements is false ?

(1) $\frac{\Delta Y}{Y}$ gets minimum contribution from the uncertainty in the length

(2) The figure of merit is the largest for the length of the rod

(3*) The maximum value of Y that can be determined is 10^{14} N/m^2

(4) $\frac{\Delta Y}{Y}$ gets its maximum contribution from the uncertainty in strain

Sol. ℓ = 1m

r = 5 × 10^{-3} m

F = 50π × 10^3 N

$\gamma = \frac{F/A}{\ell}$

$\gamma\frac{A\ell}{\ell} = \frac{F}{A}$

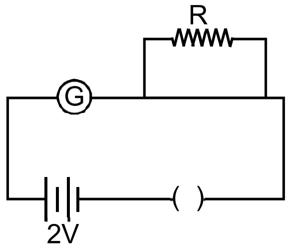
$\gamma = \frac{50\pi \times 10^3}{\pi \times (5 \times 10^{-3})^2} \times \frac{\ell}{\Delta\ell}$

$\gamma = \frac{50 \times 10^3}{25 \times 10^{-6}} \times \frac{1}{\Delta\ell} \Rightarrow \gamma = \frac{2 \times 10^9}{\Delta\ell}$

$\gamma = \frac{2 \times 10^9}{\varepsilon}$

$\gamma_{max.}$ = 2 × 10^9

27. A galvanometer has a 50 division scale. Battery has no internal resistance. It is found the there is deflection of 40 divisions when R = 2400 Ω. Deflection becomes 20 divisions when resistance taken from resistance box is 4900 Ω. Then we can conclude :



(1) Resistance of galvanometer is 200 Ω

(2) Full scale deflection current is 2mA

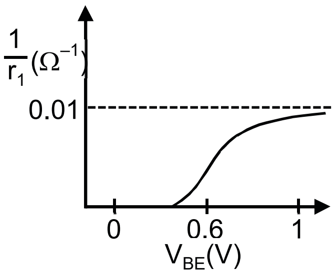
(3*) Current sensitivity of galvanometer is 20 μA/division

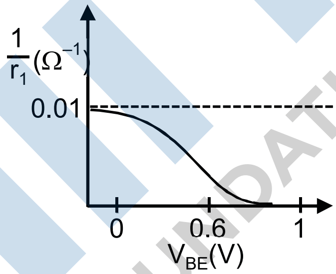
(4) Resistance required on R.B. for a deflection of 10 divisions is 9800 Ω

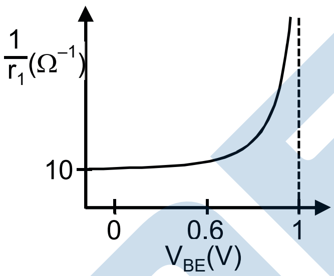
28. To determine refractive index of glass slab using a travelling microscope, minimum number of readings required are :

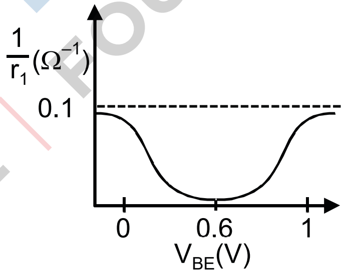
(1) Two (2*) Three (3) Four (4) Five

29. A realistic graph depicting the variation of the reciprocal of input resistance in an input characteristics measurement in a common emitter transistor configurations is :

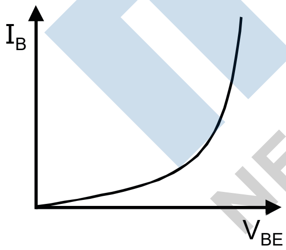
(1)


(2*)


(3)


(4)


Sol. For common emitter configuration, the input characterstics graph is as shown



$$r_i = \frac{\Delta V_i}{\Delta I_i} \Rightarrow \frac{1}{r_i} = \frac{dI_i}{dV_i} = \text{slope}$$

30. The ratio (R) of output resistance r_0, and the input resistance r_i in measurements of input and output characteristics of a transistor is typically in the range :

(1) $R \sim 10^2 - 10^3$ (2*) $R \sim 1 - 10$ (3) $R \sim 0.1 - 0.01$ (4) $R \sim 0.1 - 1.0$

Sol. $R = \frac{r_0}{r_i}$

Typical value is approximately 10

PART-B-CHEMISTRY

31. The volume of 0.1N dibasic acid sufficient to neutralize 1 g of a base that furnishes 0.04 mole of OH^- in aqueous solution is :

(1) 200 mL (2*) 400 mL (3) 600 mL (4) 800 mL

32. Initially, the root mean square (rms) velocity of N_2 molecules at certain temperature is u. If this temperature is doubled and all the nitrogen molecules dissociate into nitrogen atoms, then the new rms velocity will be:

(1) u/2 (2*) 2u (3) 4u (4) 14u

Sol. $\text{rms}(N_2) = \sqrt{\dfrac{3RT}{M_{N_2}}} = \sqrt{\dfrac{3RT}{28}} = U$ …………..(1)

After dissociation

$$\text{rms}(N) = \sqrt{\frac{3R(2T)}{M_N}} = \sqrt{\frac{3R(2T)}{14}} = 2u$$

33. Aqueous solution of which salt will not contain ions with the electronic configuration $1s^2 2s^2 2p^6 3s^2 3p^6$?

(1*) NaF (2) NaCl (3) KBr (4) CaI_2

Sol. NaF : $Na^+ = 1s^2 2s^2 2p^6$

$F^- = 1s^2\ 2s^2\ 2p^6$

34. The bond angle H-X-H is the greatest in the compound :

(1*) CH_4 (2) NH_3 (3) H_2O (4) PH_3

35. If 100 mole of H_2O_2 decompose at 1 bar and 300 K, the work done (kJ) by one mole of O_2(g) as it expands against 1 bar pressure is :

$2H_2O_2(\ell) \rightarrow 2H_2O(\ell) + O_2(g)$

$(R = 8.3\ J\ K^{-1}\ mol^{-1})$

(1) 62.25 (2*) 124.50 (3) 249.00 (4) 498.00

Sol. $2H_2O_2(I) \rightarrow 2H_2O\ (I) + O_2\ (g)$

$W = -P_{ext}\ (\Delta V) = -(\ n_{O_2}\ )\ RT$

$\because$ 100 mol H_2O_2 on decomposition will give 50 mol O_2

$\Rightarrow W = -(50)(8.3)(300)J$

$= -124500$ J

$W = -124.5$ kJ

$\Rightarrow$ Work done by O_2(g) = 124.5 kJ Ans.

36. An aqueous solution of a salt MX_2 at certain temperature has a van't Hoff factor of 2. The degree of dissociation for this solution of the salt is :

(1) 0.33 (2*) 0.50 (3) 0.67 (4) 0.80

Sol. For MX_2 type salt

Vant factor (i) = $1 + 2\alpha = 2$

$\Rightarrow \alpha = 0.5$

37. A solid XY kept in an evacuated sealed container undergoes decomposition to form a mixture of gases X and Y at temperature T. The equilibrium pressure is 10 bar in this vessel. K_p for this reaction is :

(1) 5 (2) 10 (3*) 25 (4) 100

Sol. $XY(s) \rightleftharpoons X(g) + Y(g)$

At eq. P P

Total pressure = 2P = 10 bar

$\Rightarrow P = 5$

Now, $K_P = (P_X)(P_Y) = P^2 = 25$

38. Oxidation of succinate ion produces ethylene and carbon dioxide gases. On passing 0.2 Faraday electricity through an aqueous solution of potassium succinate, the total volume of gases (at both cathode and anode) at STP (1 atm and 273 K) is :

(1) 2.24 L (2) 4.48 L (3) 6.72 L (4*) 8.96 L

39. The rate law for the reaction below is given by the expression k [A][B]

$A + B \rightarrow$ Product

If the concentration of B is increased from 0.1 to 0.3 mole, keeping the value of A at 0.1 mole, the rate constant will be :

(1) k (2*) k/3 (3) 3k (4) 9k

40. Gold numbers of some colloids are : Gelatin : 0.005 - 0.01,

Gum Arabic : 0.15 - 0.25;

Oleate : 0.04 - 1.0;

Starch : 15 - 25.

Which among these is a better protective colloid ?

(1*) Gelatin (2) Gum Arabic (3) Oleate (4) Starch

Sol. Lower the gold number, more will be protective power of colloid.

41. The following statements concern elements in the periodic table. Which of the following is true ?

(1) All the elements in Group 17 are gases.

(2) The Group 13 elements are all metals.

(3*) Elements of Group 16 have lower ionization enthalpy values compared to those of Group 15 in the corresponding periods.

(4) For Group 15 elements, the stability of +5 oxidation state increases down the group.

42. Extraction of copper by smelting uses silica as an additive to remove :

(1) Cu_2S (2*) FeO (3) FeS (4) Cu_2O

Sol. $\underset{\text{Basic impurity}}{FeO} + \underset{\text{acidic flux}}{SiO_2} \longrightarrow \underset{\text{slag}}{FeSiO_3}$

43. Identify the reaction which does not liberate hydrogen :

(1) Reaction of zinc with aqueous alkali.

(2) Electrolysis of acidified water using Pt electrodes.

(3) Allowing a solution of sodium in liquid ammonia to stand.

(4*) Reaction of lithium hydride with B_2H_6.

Sol. $B_2H_6 + 2LiH \longrightarrow 2LiBH_4$

44. The commercial name for calcium oxide is :

(1) Milk of lime (2) Slaked lime (3) Limestone (4*) Quick lime

Sol. Fact

45. Assertion : Among the carbon allotropes, diamond is an insulator, whereas, graphite is a good conductor of electricity.

Reason : Hybridization of carbon in diamond and graphite are sp^3 and sp^2, respectively.

(1) Both assertion and reason are correct, and the reason is the correct explanation for the assertion.

(2*) Both assertion and reason are correct, but the reason is not the correct explanation for the assertion.

(3) Assertion is incorrect statement, but the reason is correct.

(4) Both assertion and reason are incorrect.

Sol. Fact

46. Identify the incorrect statement :

(1) S_2 is paramagnetic like oxygen.

(2) Rhombic and monoclinic sulphur have S_8 molecules.

(3) S_8 ring has a crown shape.

(4*) The S-S-S bond angles in the S_8 and S_6 rings are the same.

Sol. Fact

47. Identify the correct statement :

(1) Iron corrodes in oxygen-free water.

(2) Iron corrodes more rapidly in salt water because its electrochemical potential is higher.

(3) Corrosion of iron can be minimized by forming a contact with another metal with a higher reduction potential.

(4*) Corrosion of iron can be minimized by forming an impermeable barrier at its surface.

Sol. Fact

48. Which of the following is an example of homoleptic complex ?

(1*) $[Co(NH_3)_6]Cl_3$ (2) $[Pt(NH_3)_2Cl_2]$ (3) $[Co(NH_3)_4Cl_2]$ (4) $[Co(NH_3)_5Cl]Cl_2$

Sol. Complex having only 1 type of ligands are examples of homoleptic complex.

49. The transition metal ions responsible for color in ruby and emerald are, respectively :

(1) Cr^{3+} and Co^{3+} (2) Co^{3+} and Cr^{3+} (3) Co^{3+} and Co^{3+} (4*) Cr^{3+} and Cr^{3+}

Sol. Fact

50. Which one of the following substances used in dry cleaning is a better strategy to control environmental pollution ?

(1) Tetrachloroethylene (2*) Carbon dioxide

(3) Sulphur dioxide (4) Nitrogen dioxide

Sol. All other gases are itself environmental pollutant

51. Sodium extract is heated with concentrated HNO_3 before testing for halogens because :

(1) Silver halides are totally insoluble in nitric acid.

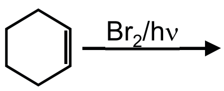
(2*) Ag_2S and AgCN are soluble in acidic medium.

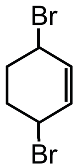
(3) S^{2-} and CN^-, if present, are decomposed by conc. HNO_3 and hence do not interfere in the test.

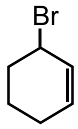
(4) Ag reacts faster with halides in acidic medium.

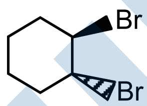
Sol. Fact

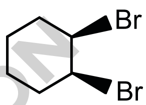
52. Bromination of cyclohexene under conditions given below yields :

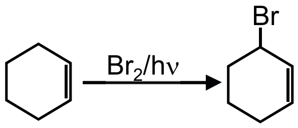


(1) 

(2*) 

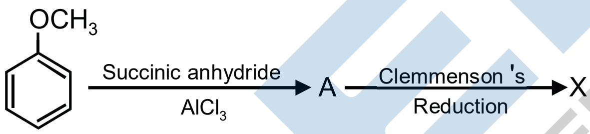
(3) 

(4) 

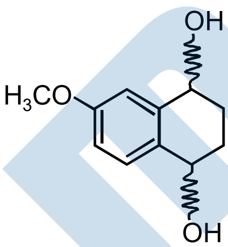
Sol. 

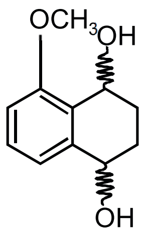
It is free radical substitution reaction.

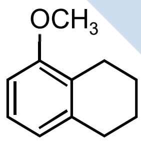
53. Consider the reaction sequence below :

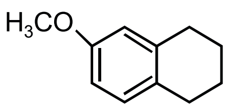


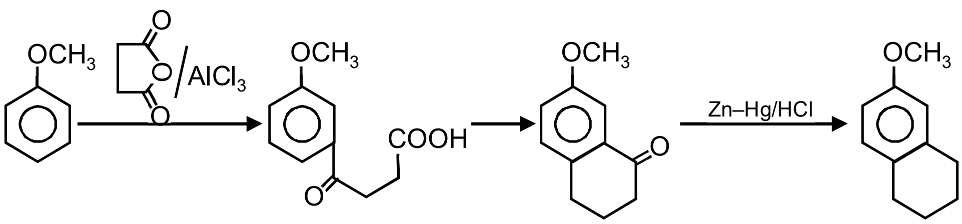
, X is :

(1) 

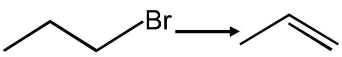
(2) 

(3) 

(4*) 

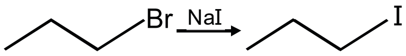
Sol. 

54. Which one of the following reagents is not suitable for the elimination reaction ?



(1) NaOH/H_2O (2) NaOEt/EtOH (3) NaOH/H_2O-EtOH (4*) NaI

Sol. NaI gives Nucleophilic substitution reaction



55. The correct statement about the synthesis of erythritol ($C(CH_2OH)_4$) used in the preparation of PETN is :

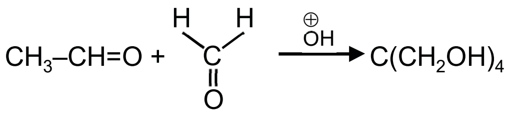
(1) The synthesis requires four aldol condensations between methanol and ethanol.

(2) The synthesis requires two aldol condensations and two Cannizzaro reactions.

(3*) The synthesis requires three aldol condensations and one Cannizzaro reaction.

(4) Alpha hydrogens of ethanol and methanol are involved in this reaction.

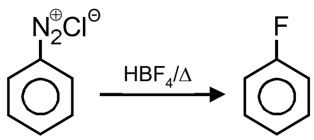
Sol. The synthesis requires three aldol & one cannizzaro reaction.



56. Fluorination of an aromatic ring is easily accomplished by treating a diazonium salt with HBF_4. Which of the following conditions is correct about this reaction ?

(1*) Only heat (2) $NaNO_2$/Cu (3) Cu_2O/H_2O (4) NaF/Cu

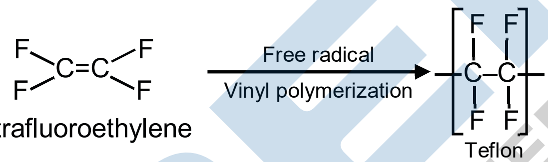
Sol.



57. Which of the following polymers is synthesized using a free radical polymerization technique ?

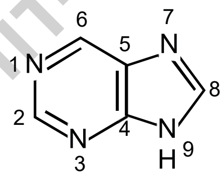
(1*) Teflon (2) Terylene (3) Melamine polymer (4) Nylon 6, 6

Sol.



Tetrafluoroethylene

58. The "N" which does not contribute to the basicity for the compound is :



(1) N 7 (2*) N 9 (3) N 1 (4) N 3

Sol. Lone pair of N9 is involve in aromaticity.

59. Which of the following is a bactericidal antibiotic ?

(1) Erythromycin (2) Tetracycline (3) Chloramphenicol (4*) Ofloxacin

Sol. All others are beteriostatic antibiotic.

60. Observation of "Rhumann's purple" is a confirmatory test for the presence of :

(1) Reducing sugar (2) Cupric ion (3) Protein (4*) Starch

Sol. Ninhydrin is often used to detect α-Amino acids and also free amino and carboxylic acid groups on proteins and peptides. Ninhydrin produce blue or purple pigment, sometimes called Ruhemann's purple.

PART-C-MATHEMATICS

61. Let P = {θ : $\sin\theta - \cos\theta = \sqrt{2}\cos\theta$} and Q = {$\theta$: $\sin\theta + \cos\theta = \sqrt{2}\sin\theta$} be two sets. Then :

(1) $P \subset Q$ and $Q - P \neq \phi$ (2) $Q \not\subset P$

(3) $P \not\subset Q$ (4*) P = Q

Sol. For Let P

$\sin\theta = \cos\theta(\sqrt{2} + 1)$

$(\sqrt{2} - 1)\sin\theta = \cos\theta$ ..... (i)

For Let Q

$\cos\theta = (\sqrt{2} - 1)\sin\theta$ .... (ii)

(i) & (ii) are same P = Q

62. If x is a solution of the equation, $\sqrt{2x+1} - \sqrt{2x-1} = 1, \left(x \geq \frac{1}{2}\right)$, then $\sqrt{4x^2 - 1}$ is equal to

(1*) $\frac{3}{4}$ (2) $\frac{1}{2}$ (3) 20 (4) $2\sqrt{2}$

Sol. $\sqrt{2x+1} = 1 + \sqrt{2x-1}$

Squaring on both sides

$2x + 1 = 1 + 2x - 1 + 2\sqrt{2x-1}$

$1 = 2\sqrt{2x-1}$

$1 = 4\sqrt{2x-1}$

x = 5/8

Now $\sqrt{4x^2 - 1}$ at x = 5/8 = $\sqrt{4x^2 - 1}$ at $x = 5/8 = \sqrt{4x\frac{25}{64} - 1} = 3/4$

63. Let z = 1 + ai be a complex number, a > 0, such that z^3 is a real number. Then the sum }
$1 + z + z^2 + ... + z^{11}$ is equal to :

(1) $-1250\sqrt{3}i$ (2) $1250\sqrt{3}i$ (3) $1365\sqrt{3}i$ (4*) $-1365\sqrt{3}i$

Sol. z = 1 + ai, a > 0

$z^3 = 1 - 3a^2 + (3a - a^3)i$ is a real number

$\Rightarrow 3a - a^3 = 0$

$\Rightarrow a^2 = 3$

$\Rightarrow a = \sqrt{3}$, a > 0

$\Rightarrow z = 1 + \sqrt{3}i$

$= 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

Now $1 + z + z^2 + ... + z^{11} = \frac{1(1 - z^{12})}{1 - z} = \frac{1 - 2^{12}(\cos 4\pi + i\sin 4\pi)}{1 - (1 + i\sqrt{3})}$

$$= \frac{1-2^{12}}{-i\sqrt{3}} = \frac{4095}{i\sqrt{3}} = -1365\sqrt{3}i$$

64. Let A be a 3 × 3 matrix such that $A^2 - 5A + 7I = O$.

Statement - I : $A^{-1} = \frac{1}{7}(5I - A)$

Statement - II : The polynomial $A^3 - 2A^2 - 3A + I$ can be reduced to $5(A - 4I)$.

Then :

(1) Statement-I is true, but Statement-II is false.

(2) Statement-I is false, but Statement-II is true.

(3*) Both the statements are true.

(4) Both the statements are false.

Sol. $A^2 - 5A + 7I = 0$ $|A| \pm 0$

$\Rightarrow \quad A - 5I = -7A^{-1}$

$\Rightarrow \quad A^{-1} = \frac{1}{7}(5I - A)$

Hence statement 1 is true

Now $A^3 - 2A^2 - 3A + I = A(A^2) - 2A^2 - 3A + I$

$= A(5A - 7I) - 2A^2 - 3A + I$

$= 3A^2 - 10A + I$

$= 5A - 20I = 3((5A - 7I) - 10A + I$

$= 5(A - 4I)$

Statement 2 also correct

65. If $A = \begin{bmatrix} -4 & -1 \\ 3 & 1 \end{bmatrix}$, then the determinant of the matrix $(A^{2016} - 2A^{2015} - A^{2014})$ is :

(1) 2014 (2) –175 (3) 2016 (4*) –25

Sol. $A = \begin{bmatrix} -4 & -1 \\ 3 & 1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} -4 & -1 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} -4 & -1 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 13 & 3 \\ -9 & -2 \end{bmatrix}$$

$$A^2 - 2A - I = \begin{bmatrix} 13 & 3 \\ -9 & -2 \end{bmatrix} - \begin{bmatrix} -8 & -2 \\ 6 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 20 & 5 \\ -15 & -5 \end{bmatrix}$$

And $|A| = -1$

$\Rightarrow \quad |A^{2016} - 2A^{2015} - A^{2014}| = |A|^{2014} \; |A^2 - 2A - I| = 1 \begin{vmatrix} 20 & 5 \\ -15 & -5 \end{vmatrix} = (-100 + 75) = -25$

66. If $\frac{^{n+2}C_6}{^{n-2}P_2} = 11$, then n satisfies the equation

(1*) $n^2 + 3n - 108 = 0$ (2) $n^2 + 5n - 84 = 0$ (3) $n^2 + 2n - 80 = 0$ (4) $n^2 + n - 110 = 0$

Sol. $\dfrac{{}^{n+2}C_6}{{}^{n-2}P_2} = 11 \quad \Rightarrow \quad \dfrac{(n+2)!}{6!(n-4)!} = 11.\dfrac{(n-2)!}{(n-4)!}$

$\Rightarrow$ (n+2)! = 11. 6! (n–2)!

$\Rightarrow$ (n+2) (n+1) n(n–1) = 11.6!

$\Rightarrow$ (n+2) (n+1) n (n–1) = 11.10.9.8

$\Rightarrow$ n + 2 = 11

$\Rightarrow$ n = 9

Which satifies the $n^2 + 3n - 108 = 0$

67. If the coefficients of x^{-2} and x^{-4} in the expansion of $\left(x^{\frac{1}{3}} + \dfrac{1}{2x^{\frac{1}{3}}}\right)^{18}$, (x > 0), are m and n respectively, then $\dfrac{m}{n}$ is equal to :

(1*) 182 (2) $\dfrac{4}{5}$ (3) $\dfrac{5}{4}$ (4) 27

Sol. $T_{r+1} = {}^{18}C_r \left(x^{1/3}\right)^{18-r} \left(\dfrac{1}{2x^{1/3}}\right)^r$

$= {}^{18}C_r \left(\dfrac{1}{2}\right)^r x^{\frac{18-2r}{3}}$

For coefficient of x^{-2}, $\dfrac{18-2r}{2} = -2 \quad \Rightarrow \quad r = 12$

For coefficient of x^{-4}, $\dfrac{18-2r}{3} = -4 \quad \Rightarrow \quad r = 15 \Rightarrow \dfrac{m}{n} = \dfrac{{}^{18}C_{12}\left(\frac{1}{2}\right)^{12}}{{}^{18}C_{15}\left(\frac{1}{2}\right)^{15}} \dfrac{{}^{18}C_6 (2)^3}{{}^{18}C_3} = 182$

68. Let $a_1, a_2, a_3, \ldots\ldots, a_n, \ldots..$ be in A.P. If $a_3 + a_7 + a_{11} + a_{15} = 72$, then the sum of its first 17 terms is equal to :

(1*) 306 (2) 153 (3) 612 (4) 204

Sol. a^1, a^2, a^3 .... a_n .... are in A.P.

$a_3 + a_{15} = a_7 + a_{11} = a_1 + a_{17} = 36$

sum of first 17 term $= \dfrac{17}{2}(a_1 + a_{17})$

$= \dfrac{17}{2} \times 36$

= 17 × 18

= 306

69. The sum $\sum_{r=1}^{10} (r^2 + 1) \times (r!)$ is equal to

(1) (11)! (2*) 10 × (11!) (3) 101 × (10!) (4) 11 × (11!)

Sol. $\sum_{r=1}^{10}(r^2+1).r!$

$=\sum_{r=1}^{10}\left\{(r+1)^2-2r\right\}r\,!$

$=\sum_{r=1}^{10}(r+1)(r+1)!-2\sum_{r=1}^{10}r.r!$

$=\sum_{r=1}^{10}\left\{(r+1)(r+1)!-r(r!)\right\}-\sum_{r=1}^{10}r.r!$

$=(11.11!-1)-\sum_{r=1}^{10}\left((r+1)!-r!\right)$

= (11.11! – 1 – (11! – 1!)

= 10.11!

70. $\lim_{x\to 0}\frac{(1-\cos 2x)^2}{2x\tan x - x\tan 2x}$ is

(1) –2 (2) $-\frac{1}{2}$ (3) $\frac{1}{2}$ (4*) 2

Sol. $\frac{\left(2\sin^2 x\right)^2}{x\left[2\left(x-\frac{x^3}{3}+\frac{2x^5}{15}.....\right)-\left(2x-\frac{8x^3}{3}....\right)\right]}$

$=\frac{4\sin^4 x}{x^4\left[-\frac{2}{3}+\frac{8}{3}\right]}=\frac{4}{2}=2$

71. Let a, b $\in$ R, (a $\neq$ 0). If the function f defined as

$$f(x)=\begin{cases}\frac{2x^2}{a} & , \quad 0\le x<1\\ a & , \quad 1\le x<\sqrt{2}\\ \frac{2b^2-4b}{x^3} & , \quad \sqrt{2}\le x<\infty\end{cases}$$

is continuous in the interval [0, ∞), then an ordered pair (a, b) is :

(1*) $\left(\sqrt{2},1-\sqrt{3}\right)$ (2) $\left(-\sqrt{2},1+\sqrt{3}\right)$ (3) $\left(\sqrt{2},-1+\sqrt{3}\right)$ (4) $\left(-\sqrt{2},1-\sqrt{3}\right)$

Sol. $$f(x)=\begin{cases}\frac{2x^2}{a} & 0\le x<1\\ a & 1\le x<\sqrt{2}\\ \frac{2b^2-4b}{x^3} & \sqrt{2}\le x<\infty\end{cases}$$

is continuous in $[0,\infty)$

$\Rightarrow$ continuous at x = 1 and x = $\sqrt{2}$

$\lim_{x\to 1^-}f(x)=\lim_{x\to 1^+}f(x)=f(1)$

$\Rightarrow \frac{2}{a} = a \Rightarrow a^2 = 2$ ....(1)

and $\lim_{x\to\sqrt{2}^-} f(x) = \lim_{x\to\sqrt{2}^+} f(x) = f(\sqrt{2})$

$\Rightarrow a = \frac{2b^2 - 4b}{2\sqrt{2}}$

$\Rightarrow b^2 - 2b = \sqrt{2}a$

If a = $\sqrt{2}$ then b2 –2b – 2 = 0 $\Rightarrow$ b = 1 ± $\sqrt{3}$

If a = – $\sqrt{2}$ then b2 – 2b + 2 = 0 $\Rightarrow$ b is imaginary which is not possible

$\Rightarrow (a,b) = (\sqrt{2}, 1+\sqrt{3}) \text{or} (\sqrt{2}, 1-\sqrt{3})$

72. Let f(x) = $\sin^4x + \cos^4x$. Then f is an increasing function in the interval :

(1) $\left]0, \frac{\pi}{4}\right[$ (2*) $\left]\frac{\pi}{4}, \frac{\pi}{2}\right[$ (3) $\left]\frac{\pi}{2}, \frac{5\pi}{8}\right[$ (4) $\left]\frac{5\pi}{8}, \frac{3\pi}{4}\right[$

Sol. f(x) = $\sin^4x + \cos^4x$

f'(x) = $4\sin^3x \cos x - 4\cos^3x \sin x$

= $4\sin x\cos x(\sin^2x - \cos^2x)$

= – 2sin2x. cos2x

= – sin4x > 0

$\Rightarrow$ sin4x < 0

$\Rightarrow \pi < 4x < 2\pi$

$\frac{\pi}{4} < x < \frac{\pi}{2}$

73. Let C be a curve given by y(x) = 1 + $\sqrt{4x-3}$, $x > \frac{3}{4}$. If P is a point on C, such that the tangent at P has slope $\frac{2}{3}$, then a point through which the normal at P passes, is :

(1) (2, 3) (2) (4, –3) (3*) (1, 7) (4) (3, –4)

Sol. $y(x) = 1 + \sqrt{4x-3},\ x, > \frac{3}{4}$

Let P $\left(\alpha, 1 + \left(\sqrt{4\alpha - 3}\right)\right)$ be the point.

at which

$\frac{dy}{dx} \text{ATP} = \frac{2}{3}$

$\Rightarrow \frac{2}{\sqrt{4\alpha - 3}} = \frac{2}{3}$

$\Rightarrow 4\alpha - 3 = 9$

$\Rightarrow \alpha = 3$ Hence P(3,4)

slope of normal at P(3,4) is $= -\frac{3}{2}$

equation of normal

$Y-4 = -\frac{3}{2}(X-3)$

2y – 8 = – 3x + 9

3x + 2y = 17

clearly it is passes through (1,7)

74. The integral $\int \frac{dx}{\left(1+\sqrt{x}\right)\sqrt{x-x^2}}$ is equal to (where C is a constant of integration)

(1) $-2\sqrt{\frac{1+\sqrt{x}}{1-\sqrt{x}}}+C$ (2*) $-2\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}+C$ (3) $-\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}+C$ (4) $2\sqrt{\frac{1+\sqrt{x}}{1-\sqrt{x}}}+C$

Sol. $I = \int \frac{dx}{\left(1+\sqrt{x}\right)\sqrt{x-x^2}}$

Put x = $\cos^2\theta$

dx = –2cosθ sinθ dθ

$I = \int \frac{-2\sin\theta\cos\theta d\theta}{(1+\cos\theta)\cos\theta\sin\theta} = -2\int \frac{d\theta}{2\cos^2\theta/2}$

$I = \int \text{sex}^2\left(\frac{\theta}{2}\right)d\theta$ $\quad \therefore \cos\theta\sqrt{x}$

$= -2\tan\theta/2 + C$ $\quad \frac{1-\tan^2\theta/2}{1+\tan^2\theta/2} = \sqrt{x}$

$= -2\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}+c \quad \Rightarrow \quad \tan^2\left(\frac{\theta}{2}\right) = \frac{1-\sqrt{x}}{1+\sqrt{x}}$

75. The value of the integral $\int_4^{10} \frac{[x^2]dx}{[x^2-28x+196]+[x^2]}$, where [x] denotes the greatest integer less than or equal to x, is :

(1) 6 (2*) 3 (3) 7 (4) $\frac{1}{3}$

Sol. $I = \int_4^{10} \frac{\left[x^2\right]dx}{\left[x^2-28x+196\right]+\left[x^2\right]}$ ....(i)

Use property $\int_a^b f(a+b-x)dx = \int_a^b f(x)dx$

$\Rightarrow I = \int_4^{10} \frac{\left[x^2-28x+196\right]dx}{\left[x^2\right]+\left[x^2-28x+196\right]}$ ....(ii)

By (i) and (ii)

$2I = \int_4^{10} dx = 10 - 4 = 6$

$I = 3$

76. For $x \in R$, $x \neq 0$, if y(x) is a differentiable function such that $x\int_1^x y(t)dt = (x+1)\int_1^x t\,y(t)dt$, then y(x) equals (where C is a constant)

(1) $\frac{C}{x}e^{-\frac{1}{x}}$ (2) $\frac{C}{x^2}e^{-\frac{1}{x}}$ (3*) $\frac{C}{x^3}e^{-\frac{1}{x}}$ (4) $Cx^3e^{\frac{1}{x}}$

Sol. $x\int_1^x y(t)dt = (x+1x)\int_1^x ty(t)dt$ ....(i)

Differentiate equation (1)

$\int_1^x y(t)dt = x^2y(x) + \int_1^x ty(t)dt$

Again differentiate

$y = 3xy + x^2\frac{dy}{dx}$

$y(1-3x) = x^2\frac{dy}{dx}$

$\frac{(1-3x)}{x^2}dx = \frac{dy}{y}$

Solve differential equation

$-\frac{1}{x} - 3\ell nx = \ell ny + \ell nc \quad x^3yc = e^{-1/x}$

$-\frac{1}{x} - \ell nx^3y + \ell nc$

$x^3yc = e^{-1/x}$

$y = \frac{c}{x^3}e^{-1/x}$

77. The solution of the differential equation $\frac{dy}{dx} + \frac{y}{2}\sec x = \frac{\tan x}{2y}$, where $0 \le x < \frac{\pi}{2}$, and y(0) = 1, is given by :

(1) $y = 1 - \frac{x}{\sec x + \tan x}$ (2) $y^2 = 1 + \frac{x}{\sec x + \tan x}$

(3*) $y^2 = 1 - \frac{x}{\sec x + \tan x}$ (4) $y = 1 + \frac{x}{\sec x + \tan x}$

Sol. $2y\frac{dy}{dx} + y^2\sec x = \tan x$

Put $y^2 = t$ $\Rightarrow$ $2y\frac{dy}{dx} = \frac{dt}{dx}$]

$$\frac{dt}{dx} + t\sec = \tan x$$

I.F. = $e^{\int \sec x dx} = e^{\ell n(\sec x + \tan x)}$ = secx + tanx

t.(secx + tanc) = $\int$ (sec x + tan x) tanx dx

$= \int \sec x \tan x dx + \int \tan^2 x dx$

y^2 (secx + tanx) = secx + tanx – x + c

y(0) = 1 $\Rightarrow$ c = 0

$$\Rightarrow \quad y^2 = 1 - \frac{x}{\sec x + \tan x}$$

78. A ray of light is incident along a line which meets another line, 7x – y + 1 = 0, at the point (0, 1). The ray is then reflected from this point along the line, y + 2x = 1. Then the equation of the line of incidence of the ray of light is :

(1*) 41x – 38y + 38 = 0 (2) 41x + 25y – 25 = 0

(3) 41x + 38y – 38 = 0 (4) 41x – 25y + 25 = 0

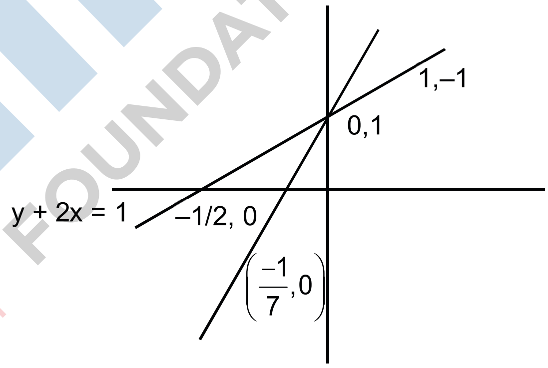
Sol. Incidene line

$L_1 + \lambda L_2 = 0$

(7x – y +1) + λ(y + 2x –1) =0

Let a point (1, –1) on y + 2x = 1

And image of (1, –1) lie on incidence line in



7x – y + 1 = 0

$$\frac{x-1}{1} = \frac{y+1}{-1} = \frac{-2(7+1+1)}{50} = x\frac{-38}{25}, \qquad y = \frac{-16}{25}$$

$$\left(7\left(\frac{-38}{25}\right) + \frac{16}{25} + 1\right) + \lambda\left(\frac{-16}{25} - \frac{76}{25} - 1\right)$$

$$\lambda = \frac{-225}{117}$$

$$(7x - y + 1)\ \frac{-225}{117}\ (y + 2x - 1) = 0$$

369x –342y + 342 = 0

41x –38y + 38 = 0

79. A straight line through origin O meets the lines 3y = 10 – 4x and 8x + 6y + 5 = 0 at points A and B respectively. Then O divides the segment AB in the ratio :

(1) 2 : 3 (2) 1 : 2 (3*) 4 : 1 (4) 3 : 4

Sol. Let equation of line thourgh 0(0, 0) is $\frac{x}{\cos\theta} = \frac{y}{\sin\theta}$ = f If this line meets 3y = 10 –4x at A then

3r, sinθ = 10–4 r_1 cosθ

r_1(3sinθ + 4cosθ) = 10…….(i)

Again the line meets 8x + 6y +5 = 0 at B

$\Rightarrow 8r_2\cos\theta + 6r_2\sin\theta + 5 = 0$

$\Rightarrow 2r_2(3\sin\theta + 4\cos) = -5$........(ii)

by $\frac{1}{2}$ $\Rightarrow$ $\frac{r_1}{2r_2} = \frac{10}{-5}$ $\Rightarrow$ $\frac{r_1}{r_2} = -\frac{4}{1} = 4$

80. Equation of the tangent to the circle, at the point (1, –1), whose centre is the point of intersection of the straight lines x – y = 1 and 2x + y = 3 is :

(1) 4x + y – 3 = 0 (2*) x + 4y + 3 = 0 (3) 3x – y – 4 = 0 (4) x – 3y – 4 = 0

Sol. Centre of circle is $\left(\frac{4}{3},\frac{1}{3}\right)$

$\Rightarrow$ equation of circle is $\left(x-\frac{4}{3}\right)^2+\left(y-\frac{1}{3}\right)^2=\left(1-\frac{4}{3}\right)^2+\left(-1-\frac{1}{3}\right)^2$

$\Rightarrow x^2-\frac{8}{3}x+\frac{16}{9}+y^2-\frac{2}{3}y+\frac{1}{9}=\frac{1}{9}+\frac{16}{9}$

$\Rightarrow x^2+y^2-\frac{8}{3}x-\frac{2}{3}y=0$ $\Rightarrow 3x^2 + 3y^2 - 8x - 2y = 0$

Equation of tangent at (1, –1) is 3x – 3y – 4(x + 1) – (y – 1) = 0

$\Rightarrow -x - 4y - 3 = 0$

$\Rightarrow x + 4y + 3 = 0$

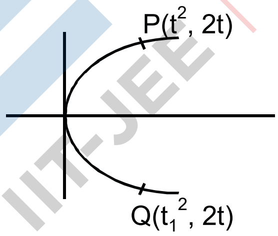
81. P and Q are two distinct points on the parabola, $y^2 = 4x$, with parameters t and t_1 respectively. If the normal at P passes through Q, then the minimum value of t_1^2 is :

(1) 2 (2) 4 (3) 6 (4*) 8

Sol. $t_1 = -t-\frac{2}{t}$

$t_1^2 = -t^2+\frac{4}{t^2}+4$

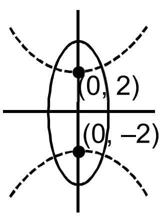
Min of t_1^2



82. A hyperbola whose transverse axis is along the major axis of the conic, $\frac{x^2}{3}+\frac{y^2}{4} = 4$ and has vertices at the foci of this conic. If the eccentricity of the hyperbola is $\frac{3}{2}$, then which of the following points does NOT lie on it ?

(1) (0, 2) (2) $\left(\sqrt{5}, 2\sqrt{2}\right)$ (3) $\left(\sqrt{10}, 2\sqrt{3}\right)$ (4*) $\left(5, 2\sqrt{3}\right)$

Sol.

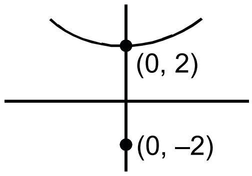


ellipse $\frac{x^2}{12}+\frac{y^2}{16}=1$

foci (0, ± be)

$e_e = \sqrt{1-\frac{12}{16}} = \frac{1}{2}$

for hyperbola



$h_H = 2$

$e_H = \frac{3}{2}$

equation $\Rightarrow \frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$

$e_H = \frac{3}{2} = \sqrt{1+\frac{a^2}{b^2}} \quad \Rightarrow \quad \frac{9}{4}-1=\frac{a^2}{b^2}$

$\frac{a^2}{b^2}=\frac{5}{4} \quad \Rightarrow \quad a^2 = 5$

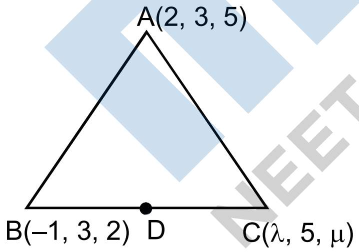
$\frac{x^2}{5}-\frac{y^2}{4}=-1$

Clearly point (5, 2 $\sqrt{3}$) does not lies on it.

83. ABC is a triangle in a plane with vertices A(2, 3, 5), B(–1, 3, 2) and C(λ, 5, μ). If the median through A is equally inclined to the coordinate axes, then the value of (λ^3 + μ^3 + 5) is :

(1) 1130 (2*) 1348 (3) 676 (4) 1077

Sol.



$D \equiv \left(\frac{-1+\lambda}{2}, 4, \frac{2+\mu}{2}\right)$

direction consine of AD $= \left\{\frac{-1+\lambda}{2}-2, 4-3, \frac{2+\mu}{2}-5\right\}$

$\left\{\frac{-1+\lambda}{2}-2, 4-3, \frac{2+\mu}{2}-5\right\}$

$\overline{AD} = \frac{\lambda-5}{2}i + j + \frac{\mu-8}{2}\hat{k}$

$$\Rightarrow \quad \frac{\left(\frac{\lambda-5}{2}\right)}{\sqrt{\left(\frac{\lambda-5}{2}\right)+1^2+\left(\frac{\mu-8}{2}\right)^2}} = \frac{1}{\sqrt{\left(\frac{\lambda-5}{2}\right)^2+1+\left(\frac{\mu-8}{2}\right)^2}} = \frac{\left(\frac{\mu-8}{2}\right)}{\sqrt{\left(\frac{\lambda-5}{2}\right)^2+1+\left(\frac{\mu-8}{2}\right)^2}}$$

$\overline{AD}.i = \overline{AD}.j = \overline{AD}.\hat{k}$

$\lambda = 7$, $\qquad \mu = 10$

$\lambda^3 + \mu^3 + 5 = 343 + 1000 + 5 = 1348$

84. The number of distinct real values of λ for which the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z+3}{\lambda^2}$ and $\frac{x-3}{1}=\frac{y-2}{\lambda^2}=\frac{z-1}{2}$ are coplanar is

(1) 4 (2) 1 (3) 2 (4*) 3

Sol. $\begin{vmatrix} 1 & 2 & \lambda^2 \\ 1 & \lambda^2 & 2 \\ 2 & 0 & 4 \end{vmatrix} = 0$

$4\lambda^2 - 2(0) + \lambda 2(-2\lambda^2) = 0$

$2\lambda^2[2 - \lambda^2] = 0$

$\lambda = 0 \quad \lambda = \pm\sqrt{2}$

85. Let ABC be a triangle whose circumcenter is at P. If the position vectors of A, B, C and P are $\vec{a}, \vec{b}, \vec{c}$ and $\frac{\vec{a}+\vec{b}+\vec{c}}{4}$ respectively, then the position vector of the orthocenter of this triangle, is :

(1) $\vec{a}+\vec{b}+\vec{c}$ (2) $-\left(\frac{\vec{a}+\vec{b}+\vec{c}}{2}\right)$ (3) $\vec{0}$ (4*) $\left(\frac{\vec{a}+\vec{b}+\vec{c}}{2}\right)$

Sol. Position vector of the centroid of ΔABC is $= \left(\frac{\vec{a}+\vec{b}+\vec{c}}{3}\right)$

Now we known that centroid divides the line joining orthocentre to circum centre divided by centriod divided by centroid in the ratio in 2 : 1

$\Rightarrow \text{orthocentre} = 3\left(\frac{\vec{a}+\vec{b}+\vec{c}}{3}\right) - 2\left(\frac{\vec{a}+\vec{b}+\vec{c}}{4}\right) = \left(\frac{\vec{a}+\vec{b}+\vec{c}}{2}\right)$

86. The mean of 5 observations is 5 and their variance is 124. If three of the observations are 1, 2 and 6 ; then the mean deviation from the mean of the data is :

(1) 2.4 (2*) 2.8 (3) 2.5 (4) 2.6

Sol. This question is wrong

$\overline{x} = \frac{x_1+x_2+x_3+x_4+x_5}{5} = 5$

$\sum_{i=1}^{5} x_i = 25$...............(i)

Also $\sigma^2 = 124$

$\Rightarrow \quad \frac{\sum x_1^2}{5} - (\bar{x}) = 124$

$\Rightarrow \quad \frac{\sum x_1^2}{5} = 124 + 25 = 149$

$\Rightarrow \quad (x_1^2 + x_2^2 + ... + x_5^2) = 745$

$\Rightarrow \quad x_1^2 + x_2^2 = 704$.................(ii)

by (i) $\quad x_1 + x_2 = 16$..................(iii)

$2x_1 x_2 = \frac{256 - 704}{2}$

$x_1 x_2 = 128 - 352 = -224$............(iv)

now $\quad \frac{\Sigma |x_1 - 5|}{5} = \frac{|x_1 - 5| + |x_2 - 5| + 4 + 3 + 1|}{5}$

$= \frac{8 + |x_1 - 5| + |11 - x_1|}{5}$

$= \frac{8 + 6}{5} = 2.8$

87. An experiment succeeds twice as often as it fails. The probability of at least 5 successes in the six trials of this experiment is :

(1) $\frac{240}{729}$ (2) $\frac{192}{729}$ (3*) $\frac{256}{729}$ (4) $\frac{496}{729}$

Sol. Given = p = 2q & we know that p+q = 1 $\Rightarrow$ P = 2/3, q = 1/3

The problem of at least 5 successes

$= {}^6C_5 P^5 q + {}^6C_6 P^6$

$= 6 \times \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right) + 1 \times \left(\frac{2}{3}\right)^6 = \frac{256}{729}$

88. If A > 0, B > 0 and A + B = $\frac{\pi}{6}$, then the minimum value of tan A + tan B is :

(1) $\sqrt{3} - \sqrt{2}$ (2) $2 - \sqrt{3}$ (3*) $4 - 2\sqrt{3}$ (4) $\frac{2}{\sqrt{3}}$

Sol. A, B > 0 and A + B = $\frac{\pi}{2}$

Let y = tanA + tanB

$\frac{dy}{dA} = \sec^2 A - \sec^2\left(\frac{\pi}{6} - A\right)$

Hence tanA + tanB $\uparrow \forall A \in \left[\frac{\pi}{12}, \frac{\pi}{6}\right]$ and tanA + tanB $\downarrow \forall A \in \left[0, \frac{\pi}{12}\right]$

clearly tanA + tanB is minimum when

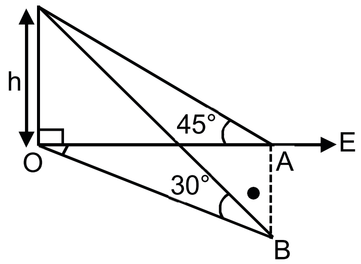
$$A = B = \frac{\pi}{12}$$

$$\Rightarrow y_{min} = 2\tan\frac{\pi}{12} = \left(2-\sqrt{3}\right)\times 2 = 4 - 2\sqrt{3}$$

89. The angle of elevation of the top of a vertical tower from a point A, due east of it is 45°. The angle of elevation of the top of the same tower from a point B, due south of A is 30°. If the distance between A and B is $54\sqrt{2}$ m, then the height of the tower (in metres), is :

(1) $36\sqrt{3}$ (2*) 54 (3) $54\sqrt{3}$ (4) 108

Sol. Let height of tower is h.



$\Rightarrow$ OA = G

$OB = \sqrt{3}h$

Also $OB^2 = OA^2 + AB^2$

$\Rightarrow 3h^2 = h^2 + (54\sqrt{2})^2 \Rightarrow h = 54$

90. The contrapositive of the following statement,

"If the side of a square doubles, then its area increases four times", is :

(1) If the side of a square is not doubled, then its area does not increase four times.

(2) If the area of a square increases four times, then its side is doubled.

(3) If the area of a square increases four times, then its side is not doubled.

(4*) If the area of a square does not increase four times, then its side is not doubled.

Sol. $p \equiv$ The side of a square doubles

$q \equiv$ Area of square increases four time

so the contrapositive of $p \to q$ is $\sim q \to \sim P$